A 'reciprocal' theorem for the prediction of loads on a body moving in an inhomogeneous flow at arbitrary Reynolds number – CORRIGENDUM

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In Magnaudet (2011) an inertial contribution was overlooked during the derivation of (3.6). Indeed, when generalizing (E.5), which is valid for an irrotational velocity field $\tilde{\boldsymbol{U}} = \nabla\tilde{\Phi}$, to (E.6) which holds for any velocity $\tilde{\boldsymbol{U}}$, an extra term $\int_{\mathscr{V}}(\tilde{\boldsymbol{U}} - \nabla\tilde{\Phi})\boldsymbol{\cdot}(\nabla\boldsymbol{U}_U + {}^T\nabla\boldsymbol{U}_U)\boldsymbol{\cdot}\hat{\boldsymbol{U}}\,\mathrm{d}V$ arises on the right-hand side of the latter and hence on that of (E.8). Therefore the right-hand side of (3.6) actually involves an additional contribution $-2\int_{\mathscr{V}}(\tilde{\boldsymbol{U}} - \nabla\tilde{\Phi})\boldsymbol{\cdot}\boldsymbol{S}_U\boldsymbol{\cdot}\hat{\boldsymbol{U}}\,\mathrm{d}V$, where $\boldsymbol{S}_U = 1/2(\nabla\boldsymbol{U}_U + {}^T\nabla\boldsymbol{U}_U)$ denotes the strain-rate tensor associated with the undisturbed flow field. This contribution to the force and torque results from the distortion by the underlying strain rate of the vortical velocity disturbance $\tilde{\boldsymbol{U}} - \nabla\tilde{\Phi}$ generated either by the dynamic boundary condition at the body surface S_B (and possibly on the wall S_W), or/and by the vorticity $\boldsymbol{\omega}_U$ of the undisturbed flow within the core of the fluid. This extra term gives in turn rise to an additional contribution $-2\int_{\mathscr{V}}(\tilde{\boldsymbol{U}}_0 - \nabla\tilde{\Phi}_0)\boldsymbol{\cdot}\boldsymbol{S}_{\boldsymbol{0}}\boldsymbol{\cdot}\hat{\boldsymbol{U}}\,\mathrm{d}V$ on the right-hand side of (3.13)–(3.15). This term was not present in the inviscid expressions established by Miloh (2003) because his derivation was restricted to situations in which the velocity disturbance is irrotational throughout the flow domain. This extra term does not alter the conclusions brought in the present paper for inviscid two-dimensional flows, nor those corresponding to the short-time limit of inviscid three-dimensional flows. In the limit $Re \to \infty$ considered in the example of § 4.3, the disturbance is still irrotational outside the boundary layers that develop around the bubble and along the wall, respectively. The leading order in the vortical velocity disturbance is of $O(Re^{-1/2})$ around the bubble, both in the $\boldsymbol{e}_\parallel$ and $\boldsymbol{e}_\perp$ directions. Near the wall, it is of $O(\kappa^2)$ (respectively $O(\kappa^2 Re^{-1/2})$) in the $\boldsymbol{e}_\parallel$ (respectively $\boldsymbol{e}_\perp$) direction. Since the thickness of both boundary layers is of $O(Re^{-1/2})$ and $\hat{\boldsymbol{U}}\boldsymbol{\cdot}\boldsymbol{e}_\perp$ grows linearly with the distance to the wall, the extra term $-\alpha\int_{\mathscr{V}}\{(\tilde{\boldsymbol{U}}_0 - \nabla\tilde{\Phi}_0)\boldsymbol{\cdot}\boldsymbol{e}_\perp(\hat{\boldsymbol{U}}\boldsymbol{\cdot}\boldsymbol{e}_\parallel) + (\tilde{\boldsymbol{U}}_0 - \nabla\tilde{\Phi}_0)\boldsymbol{\cdot}\boldsymbol{e}_\parallel(\hat{\boldsymbol{U}}\boldsymbol{\cdot}\boldsymbol{e}_\perp)\}\,\mathrm{d}V$ yields an $O(\alpha Re^{-1})$ net contribution provided by the bubble boundary layer and only an $O(\alpha\kappa^2 Re^{-1})$ correction provided by the wall boundary layer. Hence the inviscid predictions (4.26) and (4.33) are unchanged and there is an $O(\alpha Re^{-1})$ correction to the viscous drag and lift, similar in magnitude to that resulting from the term $-\int_{\mathscr{V}}\hat{\phi}\boldsymbol{\omega}_0\boldsymbol{\cdot}(\tilde{\boldsymbol{\omega}} + \tilde{\boldsymbol{\omega}}_B)_0\,\mathrm{d}V$. Owing to the weak inhomogeneity assumption invoked in §§ 3.2 and 4.3, the dimensionless shear rate α must be much smaller than unity for (4.26) and (4.33) to hold. Hence, in this context, the above corrections to the drag are much smaller than the leading, $O(Re^{-1})$, contribution provided by the surface term

$Re^{-1} \int_{S_B} \{(\hat{\boldsymbol{U}} - \mathbf{e}_\parallel) \times \boldsymbol{\omega}\} \cdot \boldsymbol{n} \, \mathrm{d}S$. Note that the above Re^{-1} prefactor is missing on the right-hand side of (4.16) and (4.19).

REFERENCES

MAGNAUDET, J. 2011 A 'reciprocal' theorem for the prediction of loads on a body moving in an inhomogeneous flow at arbitrary Reynolds number. *J. Fluid Mech.* **689**, 564–604.

MILOH, T. 2003 The motion of solids in inviscid uniform vortical fields. *J. Fluid Mech.* **479**, 279–305.